NGO & SONS

ACADEMIC COACHING

2018 HSC Mathematics Advance Solutions

Multiple Choice

Multiple Choice Answer Key

Question	Answer
1	B
2	C
3	A
4	D
5	D
6	C
7	C
8	D
9	B
10	D

Explanation

1.

$$\begin{aligned} 7^{-1.3} &= 0.079684 \\ &= 0.08 \quad (2 \text{ dp}). \end{aligned}$$

2.

$$\begin{aligned} \frac{x_p + x_Q}{2} &= x_R \\ \frac{5 + x_Q}{2} &= 9 \\ x_Q &= 18 - 5 \\ x_Q &= 13 \end{aligned}$$

$$\begin{aligned} \frac{y_p + y_Q}{2} &= y_R \\ \frac{3 + y_Q}{2} &= 5 \\ x_Q &= 10 - 3 \\ x_Q &= 7 \end{aligned}$$

$$\therefore (13, 7).$$

3.

$$x + 3y + 6 = 0.$$

x-int: let $y = 0$:

$$\therefore x = -6.$$

$$\therefore (-6, 0).$$

4.

$$\begin{aligned} r &= \frac{|\ 3(3) - 4(-2) + 3\ |}{\sqrt{3^2 + 4^2}} \\ &= \frac{20}{5} \\ &= 4 \quad \text{units.} \end{aligned}$$

$\therefore (x-3)^2 + (y+2)^2 = 16$

5.

$$\begin{aligned} y &= \sin(\ln x) \\ y' &= \frac{1}{x}\cos(\ln x) \\ \therefore y' &= \frac{\cos(\ln x)}{x}. \end{aligned}$$

6. There are 4 different pairs of shoes i.e. 8 shoes
First shoe that is picked does not matter i.e. $P = 1$
Second shoe that is picked, since there are 7 left, only 1 will match with first shoe i.e. $P = \frac{1}{7}$

$$\begin{aligned} \therefore P(matching) &= 1 \times \frac{1}{7} \\ &= \frac{1}{7} \end{aligned}$$

7.

$$\int_0^3 f(x)\,dx + \int_3^4 f(x)\,dx = \int_0^4 f(x)\,dx.$$

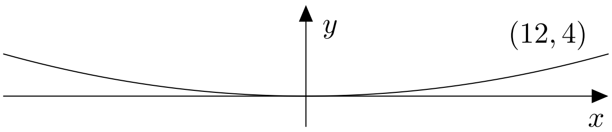
$$\begin{aligned} \implies \int_0^3 f(x)\,dx &= \int_0^4 f(x)\,dx - \int_3^4 f(x)\,dx \\ &= (10) - (-3) \\ &= 13. \end{aligned}$$

Now solving for the integral in interest, we get

$$\begin{aligned} \int_{-1}^3 f(x)\,dx &= \int_{-1}^0 f(x)\,dx + \int_0^3 f(x)\,dx \\ &= (-2) + (13) \\ &= 11 \end{aligned}$$

Hence, the answer is C.

8.



$x^2 = 4ay$

Substituting $x = 12$ and $y = 4$, we get

$12^2 = 4a(4)$
$16a = 144$
$a = 9$ cm.

9.

$x = b \Rightarrow$ stationary point of $y = f(x)$
$\therefore f''(x) = 0$

10. $\int_0^{\pi} f(x)dx = \int_{\pi}^{2\pi} f(x)dx$
$\therefore$ $function should be$ $f(x) = \cos\frac{x}{2}$

Question 11

(a)

$$\begin{aligned}\frac{3}{3+\sqrt{2}} &= \frac{3}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} \\ &= \frac{9-3\sqrt{2}}{9-2} \\ &= \frac{9-3\sqrt{2}}{7}.\end{aligned}$$

(b)

$$\begin{aligned}1-3x &> 10 \\ -3x &> 9 \\ x &< -3.\end{aligned}$$

(c)

$$\begin{aligned}\frac{8x^3-27y^3}{2x-3y} &= \frac{(2x-3y)(4x^2+6xy+9y^2)}{2x-3y} \\ &= 4x^2+6xy+9y^2.\end{aligned}$$

(d) (i)

$$T_3 = 8 \Longrightarrow 8 = a+2d$$

$$T_{20} = 59 \Longrightarrow 59 = a+19d.$$

Subtracting the above two equations, we get

$$\begin{aligned}-51 &= -17d \\ d &= 3.\end{aligned}$$

(ii)

$$T_{50} = a+49(3) = a+147.$$

From (i), $a = 8-2(3) = 2$.

$$\begin{aligned}\therefore T_{50} &= 2+49(3) \\ &= 149.\end{aligned}$$

(e)

$$\begin{aligned}\int_0^3 e^{5x}\,dx &= \left[\frac{e^{5x}}{5}\right]_0^3 \\ &= \frac{1}{5}(e^{15}-1).\end{aligned}$$

(f)

$$\begin{aligned}\frac{d}{dx}(x^2\tan x) &= 2x\tan x + \sec^2 x \cdot x^2 \\ &= x(2\tan x + x\sec^2 x).\end{aligned}$$

(g)

$$\begin{aligned}\frac{d}{dx}\left(\frac{e^x}{x+1}\right) &= \frac{e^x(x+1)-e^x}{(x+1)^2}\\ &= \frac{e^x(x+1-1)}{(x+1)^2}\\ &= \frac{xe^x}{(x+1)^2}.\end{aligned}$$

Question 12

(a) (i) $\angle ABC = 50^\circ + 60^\circ = 110^\circ$

(ii)
$$\begin{aligned} AC^2 &= AB^2 + BC^2 - 2AB \cdot BC \cdot \cos 110^\circ \\ &= 180\,089.65 \\ AC &= 424.37\ldots \\ &= 420 \text{ km (nearest 10 km)} \end{aligned}$$

(b)
$$\begin{aligned} y &= \cos 2x \\ \frac{dy}{dx} &= -2\sin 2x \\ \frac{dy}{dx}\left(x = \frac{\pi}{6}\right) &= -2\sin\frac{\pi}{3} \\ &= -2 \cdot \frac{\sqrt{3}}{2} \\ &= -\sqrt{3} \end{aligned}$$

When $x = \frac{pi}{6}$, we have $y = \cos\frac{\pi}{3} = \frac{1}{2}$. Substituting into the point gradient formula, we have

$$\begin{aligned} y - \frac{1}{2} &= -\sqrt{3}\left(x - \frac{\pi}{6}\right) \\ y &= -\sqrt{3}\left(x - \frac{\pi}{6}\right) + \frac{1}{2} \end{aligned}$$

(c) (i) In $\triangle ADF$ and $\triangle ABE$

$$\begin{aligned} AB &= AD && \text{(By definition of a square)} \\ DF &= BE \\ AC - FC &= BC - EC \\ \angle ADF &= \angle ABE \\ &= 90^\circ && \text{(Vertices of square meet at right angles.)} \\ \triangle ADF &\equiv \triangle ABE && \text{(SAS)} \end{aligned}$$

(ii) Since $EC = FC = 4$cm, then the side length of the square is 14cm. Hence, we have the area of CEF as

$$\begin{aligned} A_{CEF} &= A_S - 2 \times A_{ADF} \\ &= 14^2 - 2\left(\frac{1}{2} \cdot 14 \cdot 10\right) \\ &= 56\text{cm}^2. \end{aligned}$$

(d) (i)

$$\begin{aligned} \frac{dx}{dt} &= 2t - 4 \\ \int dx &= \int 2t - 4\,dt \\ x &= \frac{2t^2}{2} - 4t + C \end{aligned}$$

When $t = 0$, $x = 3$ and $C = 3$.

$$x = t^2 - 4t + 3.$$

(ii) When a particle is stationary, then we have

$$\begin{aligned} \frac{dx}{dt} &= 0 \\ t^2 - 4t + 3 &= 0 \\ (t-3)(t-1) &= 0. \end{aligned}$$

Hence, the particle is stationary during times $t = 1$ and $t = 3$.

(iii)

$$\frac{d^2x}{dt^2} = 2t - 4$$

Let $\frac{d^2x}{dt^2} = 0$, we have

$$\begin{aligned} 2t - 4 &= 0 \\ t &= 2. \end{aligned}$$

When $t = 2$,

$$\begin{aligned} x &= \frac{2^3}{3} - 2 \cdot 2^2 + 3 \cdot 2 \\ &= \frac{2}{3}. \end{aligned}$$

Question 13

(a) (i)

$$y = 6x^2 - x^3$$
$$y' = 12x - 3x^2$$
$$= 3x(4 - x).$$

By letting $y' = 0$, we have $x = 0$ and $x = 4$. Hence, we have stationary points at $(0, 0)$ and $(4, 32)$.

To determine the nature of the stationary points, consider the sign of y'. Here, the sign of y' is given by $y' = x\,(4 - x)$.

x		0		4	
y'	-	0	+	0	-
y	$\searrow$	0	$\nearrow$	32	$\searrow$

$\therefore$ Max at (4,32) and min at (0,0).

(ii)

$$y'' = 12 - 6x$$
$$= 6(2 - x).$$

Letting $y'' = 0$, we get

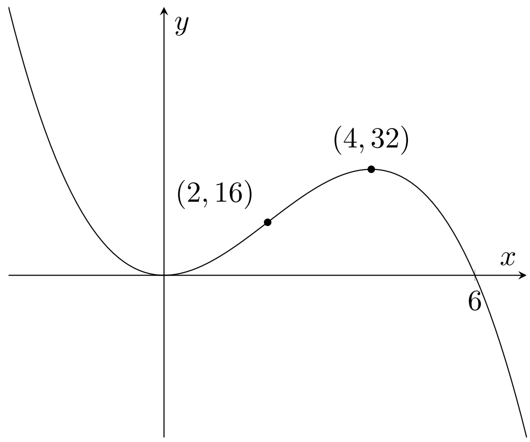
$$x = 2$$
$$\therefore y = 16.$$

Hence, the sign of y” is given by $2 - x$.

x		2	
y''	+	0	-
y	$\cup$	16	$\cap$

$\therefore$ Point of inflexion at (2,16).

(iii)



(b) (i) In $\triangle ABC$ and $\triangle CBD$

$\angle ABC$ is common

BC is common

$\angle ABC = \angle BDC$ (both $= \angle ABC$)

$\therefore \triangle ABC ||| \triangle CBD$

(ii)

$$\frac{BD}{2} = (\frac{2}{3} \quad \text{(ratios of corresponding similar triangle 's are equal for corresponding sides)}$$

$$BD = \frac{4}{3}$$

$$\therefore AD = AB - BD$$

$$= 3 - \frac{4}{3}$$

$$= \frac{5}{3}$$

(c) (i) Using the fact that in 1960 (50 years after inception), the population was 184

$$P(t) = 92e^{kt}$$

$$\text{sub } P(t) = 184, t = 50$$

$$184 = 92e^{50k}$$

$$e^{50k} = \frac{184}{92}$$

$$k = \frac{1}{50}\ln\frac{184}{92}$$

$$= 0.0139(4dp)$$

(ii) The population at time $t = 110$ is given by $P(110)$, where

$$P(100) = 92e^{\left(\frac{1}{50}\ln\left(\frac{184}{92}\right)110\right)}$$

$$= 422.72\ldots$$

$$= 423 \text{ million} \qquad \text{(nearest million)}$$

Question 14

(a) (i)

$$\begin{aligned}\triangle KLM &= \frac{1}{2}ab\sin C\\ \triangle KLM &= \frac{1}{2}(3)(6)\sin 60^\circ\\ \therefore \triangle KLM &= \frac{9\sqrt{3}}{2}\ u^2\end{aligned}$$

(ii)

$$\begin{aligned}\triangle KLM &= \triangle KLN + \triangle MLN\\ \frac{1}{2}(3)(x)\sin 30^\circ + \frac{1}{2}(6)(x)\sin 30^\circ &= \frac{9\sqrt{3}}{2}\\ \frac{3}{2}x + 3x &= 9\sqrt{3}\\ \frac{9}{2}x &= 9\sqrt{3}\\ \therefore x &= 2\sqrt{3}\ u\end{aligned}$$

(b)

$$\begin{aligned}V &= \pi\int_c^d x^2 dy\\ V &= \pi\int_1^{10}(y-1)^{\frac{1}{2}}dy\\ V &= \frac{2\pi}{3}\left[(y-1)\sqrt{y-1}\right]_1^{10}\\ V &= \frac{2\pi}{3}\left[9(3)-0\right]\\ \therefore V &= 18\pi\ u^2\end{aligned}$$

(c)

$$\begin{aligned}f(x) &= x^3 + kx^2 + 3x - 5\\ f'(x) &= 3x^2 + 2kx + 3\end{aligned}$$

To have no stationary points, there must be no solutions to $f'(x) = 0$

$$3x^2 + 2kx + 3 = 0$$

For no solutions, $\Delta < 0$

$$\begin{aligned}\Delta &= b^2 - 4ac\\ \Delta &= (2k)^2 - 4(3)(3)\\ \Delta &= 4k^2 - 36\end{aligned}$$

$$\begin{aligned}\Delta &< 0\\ 4k^2 - 36 &< 0\\ 4k^2 &< 36\\ k^2 &< 9\\ \therefore\quad -3 < k &< 3\end{aligned}$$

(d) (i)

$$T_n = 2^n + n$$

$$\begin{aligned} T_1 &= 2^{(1)} + (1) \\ &= 3 \end{aligned}$$

$$\begin{aligned} T_2 &= 2^{(2)} + (2) \\ &= 6 \end{aligned}$$

$$\begin{aligned} T_3 &= 2^{(3)} + (3) \\ &= 11 \end{aligned}$$

$\therefore T_1 = 3,\ T_2 = 6,\ T_3 = 11$

(ii)

$$\begin{aligned} S_{20} &= (2^1 + 1) + (2^2 + 2) + ... + (2^{19} + 19) + (2^{20} + 20) \\ &= (2^1 + 2^2 + ... + 2^{19} + 2^{20}) + (1 + 2 + ... + 19 + 20) \\ &= \frac{2(1 - 2^{20})}{1 - 2} + \frac{20}{2}\left[2(1) + (20 - 1)(1)\right] \qquad (*) \\ &= 2097360 \end{aligned}$$

NOTE: sum of arithmetic progression and geometric progression formula was used at $(*)$

(e) (i)

$$\begin{aligned} \text{P(at least 1 faulty pen)} &= 1 - \text{P(no faulty pens)} \\ &= 1 - \frac{9}{10} \times \frac{19}{20} \\ &= \frac{29}{200} \end{aligned}$$

(ii)

Case 1: Pick Machine A

$$\begin{aligned} \text{P(no faulty} \mid \text{A)} &= \frac{9}{10} \times \frac{9}{10} \\ &= \frac{81}{100} \end{aligned}$$

Case 2: Pick Machine B

$$\begin{aligned} \text{P(no faulty} \mid \text{B)} &= \frac{19}{20} \times \frac{19}{20} \\ &= \frac{361}{400} \end{aligned}$$

$$\begin{aligned} \therefore \text{P(no faulty)} &= \frac{1}{2}\left(\frac{81}{100}\right) + \frac{1}{2}\left(\frac{361}{400}\right) \\ &= \frac{137}{160} \end{aligned}$$

Question 15

(a) (i)

$$l(t) = 12 + 2\cos\left(\frac{2\pi t}{366}\right)$$

Substituting $t = 0$ into $l(t)$, we get

$$\begin{aligned} l(0) &= 12 + 2\cos\left(\frac{2\pi(0)}{366}\right) \\ &= 14 \end{aligned}$$

$\therefore$ The length of daylight is 14 hours.

(ii) We will find the range of $l(t)$.

$$\begin{aligned} -1 &\le \cos\left(\frac{2\pi t}{366}\right) \le 1 \\ -2 &\le 2\cos\left(\frac{2\pi t}{366}\right) \le 2 \\ 10 &\le 12 + 2\cos\left(\frac{2\pi t}{366}\right) \le 14 \\ 10 &\le l(t) \le 14 \end{aligned}$$

$\therefore$ The shortest length of daylight is 10 hours.

(iii) Let $l(t) = 11$.

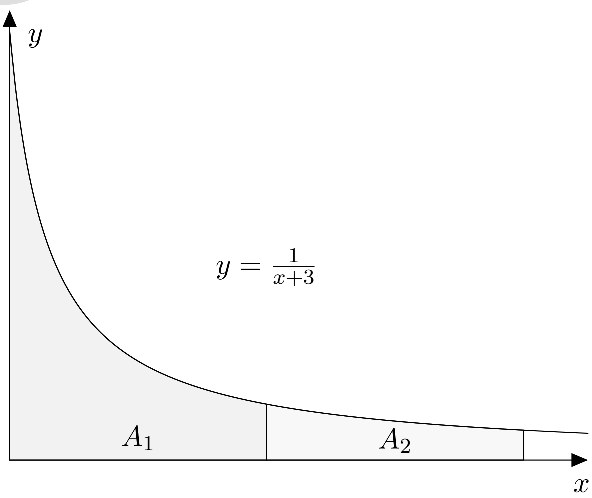
$$\begin{aligned} 12 + 2\cos\left(\frac{2\pi t}{366}\right) &= 11 \\ \cos\left(\frac{2\pi t}{366}\right) &= -\frac{1}{2} \\ \frac{2\pi t}{366} &= \pm\frac{2\pi}{3} + k2\pi \\ t &= \pm 122 + k366 \end{aligned}$$

Now for $0 \le t \le 366$, we substitute

$$k = 0 \implies t = 122$$

$$k = 1 \implies t = 244.$$

(b)



$$
\begin{aligned}
A_1 &= \int_0^k \frac{dx}{x+3} \\
&= [\ln(x+3)]_0^k \\
&= \ln(k+3) - \ln(3) \\
&= \ln\left(\frac{k+3}{3}\right).
\end{aligned}
$$

$$
\begin{aligned}
A_2 &= \int_k^{45} \frac{dx}{x+3} \\
&= [\ln(x+3)]_k^{45} \\
&= \ln(48) - \ln(k+3) \\
&= \ln\left(\frac{48}{k+3}\right).
\end{aligned}
$$

Since $A_1 = A_2$,

$$
\begin{aligned}
\ln\left(\frac{k+3}{3}\right) &= \ln\left(\frac{48}{k+3}\right) \\
\frac{k+3}{3} &= \frac{48}{k+3} \\
(k+3)^2 &= 144 \\
k+3 &= \pm 12 \\
k &= -3 \pm 12 \\
k &= 9 \quad (k > 0).
\end{aligned}
$$

(c) (i)

$$A = \int_a^b y_2 - y_1 \, dx$$

In this case, $y_2 = 2x$, $y_1 = x^3 - 7x$.

$$
\begin{aligned}
\therefore A &= \int_0^3 (2x) - (x^3 - 7x) \, dx \\
&= \int_0^3 2x - x^3 + 7x \, dx \\
&= \left[\frac{2x^2}{2} - \frac{x^4}{4} + \frac{7x^2}{2}\right]_0^3 \\
&= \frac{81}{4} \quad \text{units}^2.
\end{aligned}
$$

(ii)

$$A = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right].$$

Consider $f(x) = 2x - (x^3 - 7x) = 9x - x^3$. Now substituting values for x_i, we get

$$x_0 = 0 \implies f(x_0) = 0$$

$$x_1 = \frac{3}{2} \implies f(x_1) = 9\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2 = \frac{81}{8}$$

$$x_2 = 3 \implies f(x_2) = 9(3) - (3)^3 = 0$$

$$\begin{aligned}\therefore A &= \frac{3-0}{6}\left[0+4\left(\frac{81}{8}\right)+0\right]\\ &= \frac{81}{4} \quad \text{units}^2.\end{aligned}$$

(iii)

$$y = x^3 - 7x \implies y' = 3x^2 - 7.$$

Since parallel to $y = 2x$, we can equate gradients. Hence, we get

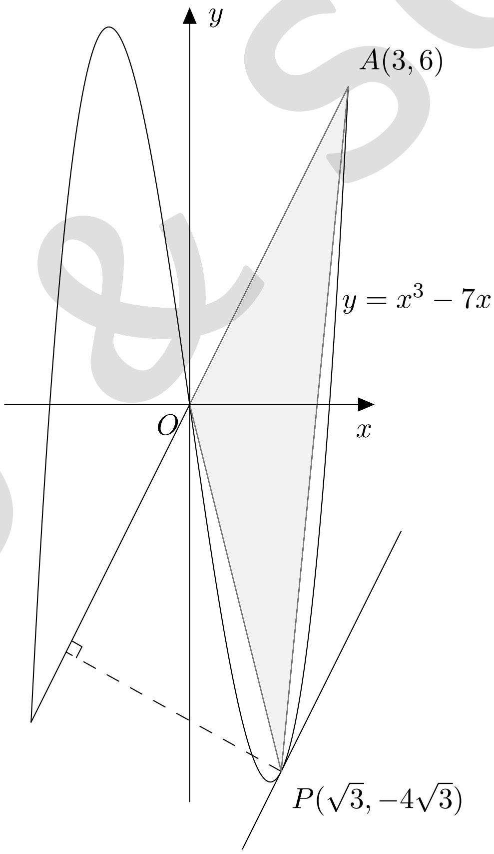
$$\begin{aligned}3x^2 - 7 &= 2\\ x^2 &= 3\\ x &= \pm\sqrt{3}\\ x &= \quad (x > 0).\end{aligned}$$

Substituting $x = \sqrt{3}$ into y,

$$\begin{aligned}y &= (\sqrt{3})^3 - 7(\sqrt{3})\\ &= -4\sqrt{3}\end{aligned}$$

$$\therefore P(\sqrt{3}, -4\sqrt{3}).$$

(iv)



$$A = \frac{1}{2}bh$$

$$\begin{aligned}b &= OA\\ &= \sqrt{(3-0)^2 + (6-0)^2}\\ &= \sqrt{45}\end{aligned}$$

$$h = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

First we will rearrange our equation to get

$$y = 2x \implies 2x - y = 0$$

and use the point $P(\sqrt{3}, -4\sqrt{3})$.

$$\begin{aligned} h &= \frac{|2(\sqrt{3}) - (-4\sqrt{3}) + 0|}{\sqrt{2^2 + (-1)^2}} \\ &= \frac{|6\sqrt{3}|}{\sqrt{5}} \\ &= \frac{6\sqrt{3}}{\sqrt{5}} \quad \text{units.} \end{aligned}$$

$$\begin{aligned} \therefore A &= \frac{1}{2}(\sqrt{45})(\frac{6\sqrt{3}}{\sqrt{5}}) \\ &= \frac{6\sqrt{135}}{2\sqrt{5}} \\ &= 9\sqrt{3} \quad \text{units}^2. \end{aligned}$$

Hence the area of ΔOAP is given by $9\sqrt{3}$ units^2.

Question 16

(a) (i)

$$V = \frac{1}{3}\pi r^2 h$$

Since the radius of the cone is x cm, $r = x$.
Let the height of the cone be h. By Pythagoras' Theorem,

$$\begin{aligned} h^2 + x^2 &= 10^2 \\ x^2 &= 100 - h^2 \\ x &= \pm\sqrt{100 - h^2} \\ \therefore x &= \sqrt{100 - h^2} \qquad \text{(since } x > 0\text{)} \end{aligned}$$

(ii)

$$\begin{aligned} V &= \frac{1}{3}\pi x^2\sqrt{100 - x^2} \\ \frac{dV}{dx} &= \frac{\pi}{3}\left(2x\sqrt{100 - x^2} + x^2\frac{-2x}{2\sqrt{100 - x^2}}\right) \\ &= \frac{\pi}{3}\left(2x\sqrt{100 - x^2} - \frac{x^3}{\sqrt{100 - x^2}}\right) \\ &= \frac{\pi}{3}\left(\frac{2x(100 - x^2) - x^3}{\sqrt{100 - x^2}}\right) \\ &= \frac{\pi}{3}\left(\frac{200x - 3x^3}{\sqrt{100 - x^2}}\right) \\ &= \frac{\pi x(200 - 3x^2)}{3\sqrt{100 - x^2}} \end{aligned}$$

(iii) Let $\frac{dV}{dx} = 0$ for stationary points.

$$\begin{aligned} \frac{\pi x(200 - 3x^2)}{3\sqrt{100 - x^2}} &= 0 \\ x(200 - 3x^2) &= 0 \\ x = 0 \text{ or } x^2 &= \frac{200}{3} \\ x &= \pm 10\sqrt{\frac{2}{3}} \end{aligned}$$

Since $x > 0$, only consider $x = 10\sqrt{\frac{2}{3}}$.
Relative maximum of V occurs when $x = 10\sqrt{\frac{2}{3}}$. However, since there are no other stationary points in the domain, $x > 0$, the global maximum of V occurs when $x = 10\sqrt{\frac{2}{3}}$.
To determine the value of θ, note that the circumference of the base of the cone is the same length as the arc subtended by the circle, radius 10 cm and angle θ.

For the circumference of the base of the cone,

$$\begin{aligned} Length &= 2\pi x \\ &= 20\pi\sqrt{\frac{2}{3}} \text{ cm} \end{aligned}$$

For the arc subtended by the circle,

$$\begin{aligned} Length &= 10\theta \\ 10\theta &= 20\pi\sqrt{\frac{2}{3}} \\ \therefore \theta &= \frac{2\sqrt{2}\pi}{\sqrt{3}} \end{aligned}$$

(b) (i) The following table indicates all combination of rolls for the first two dice.

	1	**2**	**3**	**4**	**5**	**6**
1	X	X	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$
2	X	X	X	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$
3	$\frac{1}{6}$	X	X	X	$\frac{1}{6}$	$\frac{2}{6}$
4	$\frac{2}{6}$	$\frac{1}{6}$	X	X	X	$\frac{1}{6}$
5	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	X	X	X
6	$\frac{4}{6}$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	X	X

'X' indicates no chance of winning based upon the first two roll. If winning is possible, the probability of winning on the third roll, given the first two rolls, is indicated.

$P(\text{no chance of winning}) = \frac{\text{number of X's}}{\text{total possible outcomes}} = \frac{16}{36} = \frac{4}{9}$

(ii) $P(\text{winning}) = \frac{1}{36}\left(8 \times \frac{1}{6} + 6 \times \frac{2}{6} + 4 \times \frac{3}{6} + 2 \times \frac{4}{6}\right) = \frac{5}{27}$

(c) (i)

$$\begin{aligned} A_0 &= 300000 \\ A_1 &= A_0 \times 1.04 - P \\ &= 300000(1.04) - P \\ A_2 &= A_1 \times 1.04 - 1.05P \\ &= (300000(1.04) - P(1.04) - 1.05P \\ &= 300000(1.04)^2 - 1.04P - 1.05P \\ &= 300000(1.04)^2 - P[(1.04) + (1.05)] \end{aligned}$$

(ii)

$$\begin{aligned} A_3 &= A_2 \times 1.04 - 1.05^2 P \\ &= [300000(1.04)^2 - P[(1.04) + (1.05)]](1.04) - 1.05^2 P \\ &= 300000(1.04)^3 - P[(1.04)^2 + (1.04)(1.05)] - 1.05^2 P \\ &= 300000(1.04)^3 - P[(1.04)^2 + (1.04)(1.05) + (1.05)^2] \end{aligned}$$

(iii)

$$A_n = 300000(1.04)^n - P[(1.04)^{n-1} + (1.04)^{n-2}(1.05)^1 + ... + (1.04)^1(1.05)^{n-2} + (1.05)^{n-1}]$$

Note that $[(1.04)^{n-1} + (1.04)^{n-2}(1.05)^1 + ... + (1.04)^1(1.05)^{n-2} + (1.05)^{n-1}]$ forms a geometric series of n terms, with first term is $(1.04)^{n-1}$ and common ratio $\frac{1.05}{1.04}$.

$$A_n = 300000(1.04)^n - P(1.04)^{n-1}\left(\frac{1-\left(\frac{1.05}{1.04}\right)^n}{1-\frac{1.05}{1.04}}\right)$$

For money to be in the account, $A_n > 0$.

$$\begin{aligned}
300000(1.04)^n - P(1.04)^{n-1}\left(\frac{1-\left(\frac{1.05}{1.04}\right)^n}{1-\frac{1.05}{1.04}}\right) &> 0\\
P(1.04)^{n-1}\left(\frac{1-\left(\frac{1.05}{1.04}\right)^n}{1-\frac{1.05}{1.04}}\right) &< 300000(1.04)^n\\
\frac{1-\left(\frac{1.05}{1.04}\right)^n}{1.04(1-\frac{1.05}{1.04})} &< \frac{300000}{P}\\
\frac{1-\left(\frac{1.05}{1.04}\right)^n}{-0.01} &< \frac{300000}{P}\\
1-\left(\frac{1.05}{1.04}\right)^n &> -\frac{3000}{P}\\
\left(\frac{1.05}{1.04}\right)^n &< 1+\frac{3000}{P}\\
\left(\frac{105}{104}\right)^n &< 1+\frac{3000}{P}.
\end{aligned}$$